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STABILITY OF CONTROL NEURAL NETWORKS

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ABSTRACT: In this paper, we derive a sufficient condition for stability of control neural networks in terms of certain matrix inequalities by using a discrete version of the Lyapunov second method.

Keywords: Asymptotic stability, neural networks, Lyapunov function, Delay-difference control system, Matrix inequalities.

1. INTRODUCTION

In recent decades, Hopfield neural networks have been extensively studied in many aspects and successfully applied to many fields such as pattern identifying, voice recognizing, system controlling, signal processing systems, static image treatment, and solving nonlinear algebraic system, etc. Such applications are based on the existence of equilibrium points, and qualitative properties of systems. In electronic implementation, time delays occur due to some reasons such as circuit integration, switching delays of the amplifiers and communication delays, etc. Therefore, the study of the asymptotic stability of Hopfield neural networks with delays is of particular importance to manufacturing high quality microelectronic Hopfield neural networks.

While stability analysis of continuous-time neural networks can employ the stability theory of differential system [1-3], it is much harder to study the stability of discrete-time neural networks [4] with time delays [5] or impulses [6]. The techniques currently available in the literature for discrete-time systems are mostly based on the construction Lyapunov second method [7]. For Lyapunov second method, it is well known that no general rule exists to guide the construction of a proper Lyapunov function for a given system. In fact, the construction of the Lyapunov function becomes a very difficult task.

In this paper, we consider delay-difference control system of Hopfield neural networks of the form

$$v(k+1) = -Av(k) + BS(v(k-h)) + Cu(k) + f\ , \qquad (1)$$

where $v(k) \in \Omega \subseteq \Re^n$ is the neuron state vector, $h \geq 0$, $A = diag\{a_1, \ldots, a_n\}$, $a_i \geq 0$, $i = 1, 2, ..., n$ is the $n \times n$ constant relaxation matrix, B is the $n \times n$ constant weight matrix, C is $n \times m$ constant matrix, $u(k) \in \Re^m$ is the control vector, $f = (f_1, \ldots, f_n) \in \Re^n$ is the constant external input vector and $S(z) = [s_1(z_1), \ldots, s_n(z_n)]^T$ with $s_i \in C^1[\Re, (-1,1)]$ where s_i is the neuron activations and monotonically increasing for each $i = 1, 2, ..., n$. The asymptotic stability of the zero solution of the delay-differential system of Hopfield neural networks has been developed during the past several years. We refer to monographs [8-11] and the references cited therein. Much less is known regarding the asymptotic stability of the zero solution of the delay-difference control system of Hopfield neural networks. Therefore, the purpose of this paper is to establish sufficient condition for the asymptotic stability of the zero solution of (1) in terms of certain matrix inequalities.

2. PRELIMINARIES

The following notations will be used throughout the paper. $\Re^{+}$ denotes the set of all non-negative real numbers; $\mathbf{Z}^{+}$ denotes the set of all non-negative integers; $\Re^{n}$ denotes the n-finite-dimensional Euclidean space with the Euclidean norm $\|.\|$ and the scalar product between x and y is defined by $x^T y$; $\Re^{n\times m}$ denotes the set of all $(n\times m)$-matrices; and A^T denotes the transpose of the matrix A; Matrix $Q\in\Re^{n\times n}$ is positive semidefinite $(Q\geq 0)$ if $x^T Qx\geq 0,$ for all $x\in\Re^n$. If $x^T Qx>0(x^T Qx<0$, resp.) for any $x\neq 0$, then Q is positive (negative, resp.) definite and denoted by $Q>0,(Q<0,$ resp.). It is easy to verify that $Q>0,$ $(Q<0,$ resp.) iff

$$\exists\beta>0:\ \ x^T Qx\geq\beta\|x\|^2,\forall x\in\Re^n,$$

$$(\ \exists\beta>0:\ x^T Qx\leq-\beta\|x\|^2,\forall x\in\Re^n,\ \text{resp.})\ .$$

Fact 2.1 For any positive scalar ε and vectors x and y, the following inequality holds:

$$x^T y+y^T x\leq\varepsilon x^T x+\varepsilon^{-1}y^T y\,.$$

Let us denote $V_\delta=\{x\in\Re^n:\ \ \|x\|<\delta\}.$

Lemma 2.1 [12] The zero solution of difference system is asymptotic stability if there exists a positive definite function $V(x):\Re^n\to\Re^+$ such that

$$\exists\beta>0:\Delta V(x(k))=V(x(k+1))-V(x(k))\leq-\beta\|x(k)\|^2,$$

along the solution of the system. In case the above condition holds for all $x(k)\in V_\delta$, we say that the zero solution is locally asymptotically stable.

We present the following technical lemmas, which will be used in the proof of our main result.

Lemma 2.2 [13] For any constant symmetric matrix $M\in\Re^{n\times n}, M=M^T>0$, scalar $s\in\Re^+/\{0\}$, vector function $W:[0,s]\to\Re^n$, we have

$$s\sum_{i=0}^{s-1}(w^T(i)Mw(i))\geq\left(\sum_{i=0}^{s-1}w(i)\right)^T M\left(\sum_{i=0}^{s-1}w(i)\right).$$

3. MAIN RESULTS

In this section, we consider the sufficient condition for asymptotic stability of the zero solution v^* of (1) in terms of certain matrix inequalities. Without loss of generality, we can assume that $v^*=0, S(0)=0$ and $f=0$ (for otherwise, we let $x=v-v^*$ and define $S(x)=S(x+v^*)-S(v^*))$.

The new form of (1) is now given by

$$x(k+1)=-Ax(k)+BS(x(k-h))+Cu(k). \quad (2)$$

This is a basic requirement for controller design. Now, we are interested designing a feedback controller for the system (2) as

$$u(k)=Kx(k),$$

where K is $n\times m$ constant control gain matrix.

The new form of (2) is now given by

$$x(k+1)=-Ax(k)+BS(x(k-h))+CKx(k) \quad (3)$$

Throughout this paper we assume the neuron activations $s_i(x_i)$, $i=1,2,\dots,n$ is bounded and monotonically nondecreasing on $\Re$, and $s_i(x_i)$ is Lipschitz continuous, that is, there exist constant $l_i>0, i=1,2,\dots,n$ such that

$$|s_i(r_1)-s_i(r_2)|\le l_i|r_1-r_2|, \ \forall r_1,r_2\in\Re. \quad (4)$$

By condition (4), $s_i(x_i)$ satisfy

$$|s_i(x_i)|\le l_i|x_i|, \ i=1,2,...,n. \quad (5)$$

Theorem 3.1 The zero solution of the delay-difference control system (3) is asymptotically stable if there exist symmetric positive definite matrices P,G,W and $L=diag[l_1,\dots,l_n]>0$ satisfying the following matrix inequalities of the form

$$\psi=\begin{pmatrix}(1,1) & 0 & 0\\ 0 & (2,2) & 0\\ 0 & 0 & (3,3)\end{pmatrix}<0, \quad (6)$$

where

(1,1)= $APA-APCK-K^TC^TPA-C^TK^TPC-P+hG+W+\varepsilon APBB^TPA+\varepsilon_1K^TC^TPBB^TPCK$,

(2,2)= $\varepsilon^{-1}LL+\varepsilon_1^{-1}LL+LB^TPBL-W$,

$(3,3)=-hG$.

Proof Consider the Lyapunov function $V(y(k)) = V_1(y(k)) + V_2(y(k)) + V_3(y(k))$,

where

$$V_1(y(k)) = x^T(k)Px(k),$$

$$V_2(y(k)) = \sum_{i=k-h+1}^{k}(h-k+i)x^T(i)Gx(i),$$

P, G, and W being symmetric positive definite solutions of (6) and $y(k) = [x(k), x(k-h)]$.

Then difference of $V(y(k))$ along trajectory of solution of (3) is given by

$$\Delta V(y(k)) = \Delta V_1(y(k)) + \Delta V_2(y(k)) + \Delta V_3(y(k)),$$

where

$$\begin{aligned}
\Delta V_1(y(k)) &= V_1(x(k+1)) - V_1(x(k)) \\
&= [-Ax(k) + BS(x(k-h)) + CKx(k)]^T P[-Ax(k) + BS(x(k-h)) + CKx(k)] - x^T(k)Px(k) \\
&= x^T(k)[APA - APCK - K^TC^TPA - C^TK^TPC - P]x(k) - x^T(k)APBS(x(k-h)) \\
&\quad -S^T(x(k-h))B^TPAx(k) + x^T(k)K^TC^TPBS(x(k-h)) + S^T(x(k-h))B^TPCKx(k) \\
&\quad +S^T(x(k-h))B^TPBS(x(k-h)),
\end{aligned}$$

$$\Delta V_2(y(k)) = \Delta\left(\sum_{i=k-h+1}^{k}(h-k+i)x^T(i)Gx(i)\right) = hx^T(k)Gx(k) - \sum_{i=k-h+1}^{k}x^T(i)Gx(i), \tag{7}$$

where (5) and **Fact 2.1** are utilized in (7), respectively.

Note that

$$-x^T(k)APBS(x(k-h)) - S^T(x(k-h))B^TPAx(k) \le \varepsilon x^T(k)APBB^TPAx(k) + \varepsilon^{-1}S^T(x(k-h))S(x(k-h)),$$

$$\begin{aligned}
x^T(k)K^TC^TPBS(x(k-h)) + S^T(x(k-h))B^TPCKx(k) &\le \varepsilon_1 x^T(k)K^TC^TPBB^TPCKx(k) \\
&\quad +\varepsilon_1^{-1}S^T(x(k-h))S(x(k-h)),
\end{aligned}$$

$$S^T(x(k-h))B^TPBS(x(k-h)) \le x^T(k-h)LB^TPBLx(k-h),$$

$$\varepsilon^{-1}S^T(x(k-h))S(x(k-h)) \le \varepsilon^{-1}x^T(k-h)LLx(k-h),$$

$$\varepsilon_1^{-1}S^T(x(k-h))S(x(k-h)) \le \varepsilon_1^{-1}x^T(k-h)LLx(k-h),$$

hence

$$\Delta V_1 \le x^T(k)[APA - APCK - K^TC^TPA - C^TK^TPC - P]x(k) + \varepsilon x^T(k)APBB^TPAx(k)$$
$$+\varepsilon_1 x^T(k)K^TC^TPBB^TPCKx(k) + \varepsilon^{-1}x^T(k-h)LLx(k-h) + \varepsilon_1^{-1}x^T(k-h)LLx(k-h)$$
$$+x^T(k-h)LB^TPBLx(k-h)$$

Then we have

$$\Delta V \le x^T(k)[APA - APCK - K^TC^TPA - C^TK^TPC - P + hG + \varepsilon APBB^TPA + \varepsilon_1 K^TC^TPBB^TPCK]x(k)$$
$$+x^T(k-h)[\varepsilon^{-1}LL + \varepsilon_1^{-1}LL + LB^TPBL]x(k-h) - \sum_{i=k-h}^{k-1} x^T(i)Gx(i).$$

Using **Lemma 2.2**, we obtain

$$\sum_{i=k-h+1}^{k} x^T(i)Gx(i) \ge \left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right)^T (hG)\left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right).$$

From the above inequality it follows that:

$$\Delta V \le x^T(k)[APA - APCK - K^TC^TPA - C^TK^TPC - P + hG + \varepsilon APBB^TPA + \varepsilon_1 K^TC^TPBB^TPCK]x(k)$$
$$+x^T(k-h)[\varepsilon^{-1}LL + \varepsilon_1^{-1}LL + LB^TPBL]x(k-h) - \left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right)^T (hG)\left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right)$$
$$= \left(x^T(k), x^T(k-h), (\frac{1}{h}\sum_{i=k-h+1}^{k} x(i))^T\right)\begin{pmatrix}(1,1) & 0 & 0\\ 0 & (2,2) & 0\\ 0 & 0 & (3,3)\end{pmatrix}\begin{pmatrix}x(k)\\ x(k-h)\\ (\frac{1}{h}\sum_{i=k-h+1}^{k} x(i))\end{pmatrix}$$
$$= y^T(k)\psi y(k),$$

where

$$(1,1) = APA - APCK - K^TC^TPA - C^TK^TPC - P + hG + \varepsilon APBB^TPA + \varepsilon_1 K^TC^TPBB^TPCK,$$
$$(2,2) = \varepsilon^{-1}LL + \varepsilon_1^{-1}LL + LB^TPBL,$$
$$(3,3) = -hG,$$
$$y(k) = \begin{pmatrix}x(k)\\ x(k-h)\\ (\frac{1}{h}\sum_{i=k-h+1}^{k} x(i))\end{pmatrix}.$$

By the condition (6), $\Delta V(y(k))$ is negative definite, namely there is a number $\beta > 0$ such that $\Delta V(y(k)) \leq -\beta \|y(k)\|^2$, and hence, the asymptotic stability of the system immediately follows from **Lemma 2.1**. This completes the proof. □

4. CONCLUSION

In this paper, based on a discrete analog of the Lyapunov second method, we have established a sufficient condition for the stability of control neural networks in terms of certain matrix inequalities.

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