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OPERATIONS IN ALGEBRA

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Abstract.Algebra is the study of the structure of certain sets along with operations on them. An operator is a special character or combination of characters that operates on variables .In this paper we have studied the definitions of unary, binary and ternary operations and have given some examples.

Keywords: Unary, binary and ternary operations.

INTRODUCTION

Modern Algebra is the study of the structure of certain sets along with operations on them.The algebras discussed here are semigroup, groups, rings, near-ring, ternary semigroup, ternary semirings and boolean algebras. They are useful throughout mathematics and computer science.An algebra consists of two components, i)A set A (say) which is non-empty and ii) operators defined on A.Each operator is a function of the type

$$A^n \to A \text{ for some n, where } A^n = A \times \underbrace{A \times A \times A \ldots\ldots\times A}_{\text{n times}}$$

An operator is a special character or combination of characters that operates on variables. In this paper we have discussed about unary, binary and ternary operations. A unary operation is an operation with only one element (called operand). A binary operation on a set is a calculation that combines two elements of the set to produce another element of the set. Ternary and n-ary generalizations of algebraic structures are the most natural ways for further development and deeper understanding of their fundamental properties. Firstly, ternary algebraic operations were introduced already in thenineteenth century by A. Cayley. As the development of Cayley's ideas it were considered n-ary generalization of matrices and their determinants and general theory of n-ary algebras.

UNARY OPERATION

A unary operation is an operation with only one operand, i.e. a single input.

Definition 2.1:-Let A be a non-empty set. A function $f : A \to A$ is a unary operation on A.

Now we quote some examples of binary operations.

Examples 2.2:-

i) The operation $f : A \to A$ defined by $f(a) = -a$, for all $a \in A$ is a unary operation on $A = Z$, Q, R.

ii) Let$X$ be a non-empty set and P(X) be the set of all subsets of X. An operation f defined by $f(A) = A'$ for all $A \in \mathrm{P}(X)$ is a unary operation on $\mathrm{P}(X)$, where A' is the complement of A.

iii) Let $M_2(R)$ be a set of all 2 × 2 non-singular matrices. The operation $f : M_2(R) \to M_2(R)$ defined by

$f(X) = X^{-1}$, for all $X \in M_2(R)$ is a unary operation on $M_2(R)$, where X^{-1} is the multiplicative inverse of

X.

iv) Let N be the set of natural numbers. The operation $f : N \to N$ defined by $f(a) = a + 1$, for all $a \in N$ is a

unary operation on N.

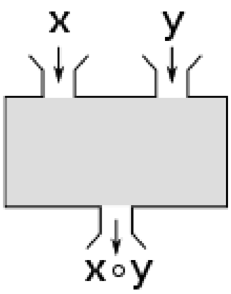
BINARY OPERATION

Definition 3.1: -Let A be a non-empty set. A binary operation on A is a function that assigns each ordered pair

of elements of A to an element of A.

A function $f : A \times A \to A$ where A is a non-empty set. The function f is called a binary operation on A.

Diagrammatic Representation of Binary Operation



The word "binary" means composed of twopieces

Binary operations are the keystone of algebraic structures studied in an Abstract Algebra they are essential in the definitions of groups, monoids, semigroups, semirings,rings, near-rings and more.

Now we quote some examples of binary operations.

Examples 3.2:-

i) Let Z be the set of all integers. The most familiar binary operations are ordinary addition, subtraction and

multiplication of integers. Division of integers is not a binary operation on the integers (because an integer

divided by an integer need not be an integer, as $2,3 \in Z$, but $2/3 \in Z$).

ii) Let X be a non-empty set and $\mathrm{P}(X)$ be the set of all subsets of X. An operation $*$ defined by

$A * B = A \cup B$ for all $A, B \in \mathrm{P}(X)$is a binary operation on $\mathrm{P}(X)$.

iii) Let$X$ bea non-empty set and $\mathrm{P}(X)$ be the set of all subsets of X. An operation $*$ defined by

$A * B = A \cap B$ for all $A, B \in \mathrm{P}(X)$is a binary operation on $\mathrm{P}(X)$.

iv) Let $M_2(R)$be a set of all 2×2 matrices. An operation $*$ defined by

$A_1 * A_2 = A_1 + A_2 \forall A_1, A_1 \in M_2(R)$,is a binary operation on $M_2(R)$.

v) Let $M_2(R)$ be a set of all 2×2 matrices. An operation $*$ defined by

$A_1 * A_2 = A_1 A_2 \;\; \forall A_1, A_1 \in M_2(R)$,is a binary operation on $M_2(R)$.

vi) Let G be anon-empty set. An operation $\times$ defined by $G \times G = \{(a, b): a \in G, b \in G\}$, is a binary operation on G.

vii) Let C be a nonempty set and S be the set of all functions $f: C \to C$. Define $\Phi: S \times S \to S$ by

$\Phi(f_1, f_2)(c) = f_1 {}^\circ f_2(c) = f_1(f_2(c))$ for all $c \in C$, the composition of the two functions $f_1$and $f_2$in S.

Then Φ is a binary operation, since the composition of the two functions is another function on the

set C.

viii) Let S be a set, and let $\mathrm{P}(S)$ be the set of all subsets of S. An operation Δ defined by

$A \Delta B = (A \setminus B) \cup (B \setminus A)$ for all $A, B \in \mathrm{P}(S)$, is a binary operation on $\mathrm{P}(S)$.

ix) Let R be the set of real numbers. An operation $\vee$ defined by $x \vee y = \max(x, y)$for all $, y \in R$, is a binary

operation on R.

x) Let R be the set of real numbers. An operation $\wedge$ defined by $x \wedge y = \min(x, y)$for all $, y \in R$, is a binary

operation on R.

xi) Let $= \{0, a, b, c, x, y\}$. An operation + on N be defined as in Table (3.1) is a binary operation on N.

+	0	a	b	c	x	y
0	0	a	b	c	x	y
a	a	0	y	x	c	b
b	b	x	0	y	a	c
c	c	y	x	0	b	a
x	x	b	c	a	y	0
y	y	c	a	b	0	x

(Table 3.1)

xii) Consider the set of integer modulo 8, $\mathbb{Z}_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Then $\mathbb{Z}_8$ is closed with respect to a

binary operation $+_8$, where$a +_8 b$ = the least non- negative remainder when $a + b$ is divided by 8 .

TERNARY OPERATION

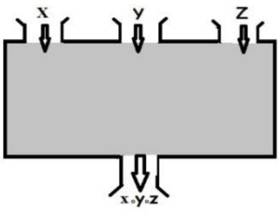
In mathematics, a ternary operation is an *n*-ary operation with *n* = 3. A ternary operation on a set *A* takes any given three elements of A and combines them to form a single element of A.

Ternary operations are the keystone of algebraic structures studied in ternary algebraic systems,they are essential in the definitions of ternary groups, ternary semigroups,ternary semirings, ternaryrings, ternary near-rings and more.

Definition 4.1:-Let *A* be a non-empty set. A ternary operation on *A* is a function that assigns each ordered triplet of elements of *A* to an element of *A*.

A function $f: A\times A \times A \rightarrow A$, where *A* is a non-empty set. Then the function f is called a ternary operation on *A*.

Diagrammatic Representation of Ternary Operation:-



Remark 4.2:-Every binary operation ( ) on a non-empty set A is a ternary operation [ ] by putting $[abc] = \big((ab)c\big) = (a(bc))$ for all $a, b, c \in A$. But every ternary operation on a non-empty set A need not be a binary operation. We illustrate this by using following examples.

i) Let $\mathbb{Z}^-$ be the set of all negative integers. An operation [ ] defined by $[abc] = \big((ab)c\big) = (a(bc))$ for all $a, b, c \in \mathbb{Z}^-$, where ( ) is usual multiplication, is a ternary operation on $\mathbb{Z}^-$, but $\mathbb{Z}^-$ is not closed with respect to binary multiplication.

ii) Consider a set $T = \{-i, 0, i\}$. T is closed with respect to ternary multiplication over complex number while

T is not closed with respect to binary multiplication over complex numbers.

Now we quote someexamples of ternary operations.

Examples 4.3:-

i)Let$\mathbb{Z}^-$ be the set of all negative integers. An operation [ ] defined by$[abc] = \big((ab)c\big) = (a(bc))$ for all $a, b, c \in \mathbb{Z}^-$, where ( ) is usual multiplication, is a ternary operation on $\mathbb{Z}^-$.

ii) Let $T = \{\, ni : n \in Z \,, i$ is an imaginary unit$\}$, where Z is the set of all integers. T is closed with respect to ternary multiplication over complex number.

iii) Let M be the set of all 2×2 matrices over the set of all integers. An operation [ ] defined by $[ABC] = \big((AB)C\big) = \big(A(BC)\big)$ for all $A, B, C \in M$, where ( ) is usual matrix multiplication, is a ternary operation on M.

iv) Let $T = \{0, a, b, c\}$. An operation [ ] by $[abc] = \big((ab)c\big) = (a(bc))$, for all $a, b, c \in T$ where ( ) is defined by the following table(Table4.1):

( )	0	a	b	c
0	0	0	0	0
a	0	a	b	a
b	0	b	0	b
c	0	c	b	c

(Table 4.1)

is a ternary operation on T.

v) Let X be a non-empty set and $P(X)$ be the set of all subsets of X. An operation $*$ defined by $A * B * C = A \cap B \cap C$ for all $A, B, C \in P(X)$ is a ternary operation on $P(X)$.

vi) Let T be a non-empty set. Define an operation [ ] on T by $[abc] = a,$ for all $a, b, c \in T$. Then [ ] is a ternary operation on T.

vii) Let M be the set of all 2×2 matrices over the set of all integers. An operation [ ] defined by $[ABC] = \big((AB)C\big) = \big(A(BC)\big)$ for all $A, B, C \in M,$ where ( ) is usual matrix multiplication, is a ternary operation on M.

viii) Let $N = \{0, a, b, c, x, y\}$.An operation [ ] be defined as $[xyz] = (x \cdot y) \cdot z$ for all $x, y, z \in N,$ where defined as in Table (ii) , is a ternary operation on N.

.	0	a	b	c	x	y
0	0	0	0	0	0	0
a	0	a	a	a	0	0
b	0	a	a	a	0	0
c	0	a	a	a	0	0
x	0	0	0	0	0	0
y	0	0	0	0	0	0

(Table 4.2)

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