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FUZZY ANTI 2 NORMED LINEAR SPACE

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ABSTRACT

In this paper , we study fuzzy anti 2-norm on a linear space and some results are introduced in fuzzy anti 2-norm on a linear space. We shall introduce the notion of convergent sequence, Cauchy sequence in fuzzy anti 2-normed linear space also introduce the concept of compact subset and bounded subset in fuzzy anti 2-normed linear space. Then we introduced closed graph theorem and proved it in Fa 2-NLS and Riesz theorem in Fa n-NLS. Then we study the concept of t –best coapproximation in fuzzy anti 2-normed linear space.

Keywords*: Fuzzy anti 2-norm, convergent sequence, Cauchy sequence, Fuzzy anti 2-linear operator.*

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1. BASIC DEFINITIONS IN FUZZY ANTI-2 NORMED LINEAR SPACE

Definition 1.1:

A function $\sigma : A \to [0,1]$ is called a membership function on *A*. The set *A* together with a membership function σ is called a fuzzy set and is denoted by (A,σ). we can also denote this *A* as a *fuzzy set* or σ is a fuzzy set.

Definition 1.2:

A *normed linear space* is a vector space *X* and a non-negative valued mapping $\|\cdot\|$ on *X*, called the norm , which satisfies the properties

1. **||x||=0 if and only if x=0.**
2. **||a x|| =|a| ||x||, for all scalars a.**
3. **||x+y|| ≤ ||x|| + ||y||**

Definition 1.3:

Let *X* be a linear space over a real field *F* (field of real/complex numbers). A fuzzy subset *N* of $X \times R$ is called a fuzzy norm on *X* if the following conditions are satisfied for all $x, y \in X$

F1: **For all $t \in R$ with $t \le 0$, $N(x, t) = 0$**

F2: **For all $t \in R$ with $t>0$, $N(x, t) = 1$ if and only if $x = \underline{0}$**

F3: **For all $t \in R$ with $t>0$, $N(cx, t) = N(x, \frac{t}{|c|})$ if $c \ne 0$, $c \in F$**

F4: **For all $s, t \in R$, $N(x + y, s + t) \ge \min\{N(x, s), N(y, t)\}$**

F5: **$N(x, .)$ is a non-decreasing function of R and $\lim_{t\to\infty} N(x,t) = 1$.**

Then N is said to be a *fuzzy norm on a linear space* X and the pair (X, N) is said to be a Fuzzy normed linear space (FNLS).
The following condition of fuzzy norm N will be required later on

F6: $N(x, t) > 0$, $\forall\, t > 0$ **implies** $x = \underline{0}$

Definition 1.4:

Let U be a linear space over a real field F. A fuzzy subset N^* of $X\times R$ such that for all $x, u \in U$ and $c \in F$

N^*1: **For all $t \in R$ with $t \le 0$, $N^*(x, t)=1$,**

N^*2: **For all $t \in R$ with $t>0$** $N^*(x,t) = 0 \Leftrightarrow x = \underline{0}$

N^*3: **For all $t \in R$ with $t>0$, $N^*(cx,t) = N^*(x, \frac{t}{|c|})$ if** $c \neq 0, c \in F$

N^*4: **For all $s, t \in R$, $N^*(x+ u, s+ t) \le \max\{N^*(x, s), N^*(u, t)\}$**

N^*5: **$N^*(x, t)$ is a non-increasing function of $t \in R$ and $\lim_{t\to\infty} N^*(x, t) = 0$.Then N^* is said to be a *fuzzy anti norm on a linear space* U and the pair (U, N^*) is called a fuzzy anti normed linear space (briefly Fa-NLS).**

N^*6: **For all $t \in R$ with $t>0$,** $N^*(x,t) < 1$ **implies** $x = \underline{0}$

Definition 1.5:

Let U be a linear space over a real field F. A fuzzy subset N^* of $U\times U\times R$ such that for all $x, y, u \in U$

N1: For all $t \in R$ with $t \le 0$, $N^*(x, y, t)=1$.

N2: For all $t\in R$ with $t>0$, $N^*(x, y, t) = 0$ if and only if x, y are linearly dependent.

N3: $N^*(x, y, t)$ is invariant under any permutation of x, y.

N4: For all $t \in R$ with $t>0$, $N^*(x, cy, t) = N^*(x, y, \frac{t}{|c|})$ if $c \neq 0, c \in F$.

N5: For all $s, t \in R$, $N^*(x, y+u, s+t) \le \max\{N^*(x, y, s), N^*(x, u, t)\}$.

N6: $N^*(x, y, t)$ is a non-increasing function of $t \in R$ and $\lim_{t\to\infty} N^*(x, y, t) = 0$.

Then N^* is said to be a fuzzy anti-2-norm on a linear space U and the pair (U, N^*) is called a *fuzzy anti-2-normed linear space*.(briefly Fa-2-NLS).

The following condition of fuzzy anti-2-norm N^* will be required later on.

N7: For all $t \in R$ with $t>0$, $N^*(x, y, t) < 1$, implies that x, y are linearly dependent.

Example :

Let (U, •, •) be a 2-normed linear space .Define

$$N^*(x,y,t) = \frac{\|x,y\|}{t+\|x+y\|}, \text{ when } t>0,\ t \in R,\ x, y \in U$$

$$= 1, \text{ when } t \le 0,\ t \in R,\ x, y \in U.$$

Then (U, N^*) is an Fa-2-NLS.

Proof:

Now we have to show that $N^*(x, y, t)$ is a fuzzy anti-2-norm in U

N1: For all $t \in R$ with $t \le 0$, we have by definition $N^*(x, y, t)=1$.

N2: For all $t \in R$ with $t>0$ ~~$N^*(x,$~~ $y, t)=0 \Leftrightarrow \frac{\|x,y\|}{t+\|x,y\|}=0 \Leftrightarrow \|x,y\|=0 \Leftrightarrow$ x, y are linearly independent.

N3: As $\|x,y\|$ is invariant under any permutation of x, y it follows that $N^*(x,y,t)$ is invariant under any permutation of x, y.

N4: For all $t \in R$ with $t>0$ and $c \neq 0$, $c \in F$, we get

$$N^*(x,cy,t)=\frac{\|x,cy\|}{t+\|x,cy\|}=\frac{|c|\|x,y\|}{t+|c|\|x,y\|}=\frac{\|x,y\|}{t/|c|+\|x,y\|}=N^*(x,y,\ t/|c|)$$

N5: For all $s, t \in R$ and $x, y, u \in U$. We have to show that $N^*(x, y+u, s+t) \le \max\{N^*(x,y,s), N^*(x,u,t)\}$. If (a) $s+t<0$ (b) $s=t=0$ (c) $s+t>0$; $s>0, t<0$; $s<0, t>0$, then in these cases the relation is obvious . If (d) $s>0, t>0, s+t>0$. Then assume that

$$N^*(x,y,s)\le N^*(x,u,t)\Rightarrow \frac{\|x,y\|}{s+\|x,y\|}\le\frac{\|x,u\|}{t+\|x,u\|}\Rightarrow \|x,y\|(t+\|x,u\|)\le\|x,u\|(s+\|x,y\|)$$

$$\Rightarrow t\|x,y\|\le s\|x,u\| \quad ....(1)$$

Now,

$$\frac{\|x,y+u\|}{s+t+\|x,y+u\|}-\frac{\|x,u\|}{t+\|x,u\|}\le\frac{\|x,y\|+\|x,u\|}{s+t+\|x,y\|+\|x,u\|}-\frac{\|x,u\|}{t+\|x,u\|}=\frac{t\|x,y\|-s\|x,u\|}{(s+t+\|x,y\|+\|x,u\|)(t+\|x,u\|)}$$

By using equation (1), we get $\frac{\|x,y+u\|}{s+t+\|x,y+u\|}\le\frac{\|x,u\|}{t+\|x,u\|}$, similarly

$$\frac{\|x,y+u\|}{s+t+\|x,y+u\|}\le\frac{\|x,y\|}{s+\|x,y\|}$$

Hence $N^*(x,y+u,s+t)\le\max\{N^*(x,y,s),N^*(x,u,t)\}$

N6: If $t_1<t_2\le 0$ then we have $N^*(x,y,t_1)=N^*(x,y,t_2)=1$. If $0<t_1<t_2$ then

$$\frac{\|x,y\|}{t_1+\|x,y\|}-\frac{\|x,y\|}{t_2+\|x,y\|}=\frac{\|x,y\|(t_2-t_1)}{(t_1+\|x,y\|)(t_2+\|x,y\|)}>0\Rightarrow N^*(x,y,t_1)\ge N^*(x,u,t_2)$$

Thus $N^*(x,y,t)$ is a non-increasing function of $t \in R$. Again

$$\lim_{t\to\infty}N^*(x,y,t)=\lim_{t\to\infty}\frac{\|x,y\|}{t+\|x,y\|}=0\ \forall x,y\in U \ .$$

Hence (U,N^*) is an Fa-2 NLS.

Definition 1.6:

Let N^* **be a fuzzy anti-2-norm on** U **satisfying N7. Define** $\|x,y\|_\alpha^* = \inf\{t>0: N^*(x,y,t)<\alpha, \alpha\in(0,1]\}$.

Lemma 1.7:

Let (U,N^*) **be a Fa-2-NLS. For each** $\alpha \in (0,1]$ **and** $x, y, u \in U$. **Then we ha**

(i) $\|x,y\|_{\alpha_1}^* \geq \|x,y\|_{\alpha_2}^*$ **for** $0<\alpha_1<\alpha_2\leq 1$,

(ii) $\|x,cy\|_\alpha^* = |c|\|x,y\|_\alpha^*$ **for any scalar** c,

(iii) $\|x,y+u\|_\alpha^* \leq \|x,y\|_\alpha^* + \|x,u\|_\alpha^*$

Proof:

(i) For $0<\alpha_1<\alpha_2\leq 1$, **we note that**

$$\inf\{t>0: N^*(x,y,t)<\alpha_1\} \geq \inf\{N^*(x,y,t)<\alpha_1\} \Rightarrow \|x,y\|_{\alpha_1}^* \geq \|x,y\|_{\alpha_2}^*$$

(ii) For any scalar c **and for all** $\alpha \in (0,1]$,

$$\|x,cy\|_\alpha^* = \inf\{t>0: N^*(x,cy,t)<\alpha, \alpha\in(0,1]\} = \inf\{t>0: N^*(x,\bar{y},\frac{t}{|c|})<\alpha, \alpha\in(0,1]\}$$

$$= |c|\inf\{t>0: N^*(x,y,t)<\alpha, \alpha\in(0,1]\} = |c|\|x,y\|_\alpha^*$$

(iii) For any $\alpha \in (0,1]$,

$$\|x,y\|_\alpha^* + \|x,u\|_\alpha^* = \inf\{t>0: N^*(x.y,t)<\alpha\} + \inf\{s>0: N^*(x,u,s)<\alpha\}$$

$$\geq \inf\{s+t>0: N^*(x,y,t)<\alpha, N^*(x,u,s)<\alpha\} = \|x,y+u\|_\alpha^*$$

Theorem 1.8.

Let $\{\|\bullet,\bullet\|_\alpha^* : \alpha\in(0,1]\}$ **be a decreasing family of 2-norms on a linear space** U. **Now define a function**

$N_1^*: U\times U\times R \to [0,1]$ **as**

$$N_1^*(x,y,t) = \inf\{\alpha\in(0,1]: \|x,y\|_\alpha^*\}, \text{ when } (x,y,t)\neq 0$$
$$= 1, \text{ when } (x,y,t) = 0$$

Then,

(a) N^* **is a fuzzy anti-2-norm on** U.

(b) For each $x, y \in U$, $\exists\, r = r(x,y)>0$ **such that** $N_1^*(x,y,t) = 1$

Proof.

(a) Now we have to show that N^* **is a fuzzy anti-2-norm on** U.

N1: (i) For all $t \in R$ **with** $t<0$, $\{\alpha\in(0,1]: \|x,y\|_\alpha^* \leq t\} = \Phi, \forall\ x,y\in U$, **we have**

$$N_1^*(x,y,t) = \inf\{\alpha\in(0,1]: \|x,y\|_\alpha^* \leq t\} = 1.$$

(ii) For $t=0$ **and** $x\neq \underline{0}$, $y\neq \underline{0}$, $\{\alpha\in(0,1]: \|x,y\|_\alpha^* \leq t\} = \Phi$, $\forall\ x,y\in U$, **we have** $N^*(x,y,t) = 1$.

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(iii) For t=0 and $x \neq \underline{0}$, $y \neq \underline{0}$, then from the definition $N_1^*(x, y, t)$= 1.

Thus for all $t \in R$ with $t \le 0$, $N_1^*(x, y, t)$=1, $\forall x, y \in U$.

N2: For all $t \in R$ with t>0, $N^*(x, y, t)$ = 0. Choose any ε ∈ (0,1). Then for any t>0, $\exists \alpha_1 \in (\varepsilon,1]$ such that $\|x.y\|^*_{\alpha_1} \le t$ and hence $\|x,y\|^*_{\varepsilon} \le t$. Since t>0 is arbitrary, this implies that $\|x,y\|^*_{\varepsilon} = 0$ then x, y are linearly independent. If x, y are linearly dependent then for t>0,

$N_1^*(x,y,t) = \inf\{\alpha \in (0,1] : \|x,y\|^*_{\alpha} \le t\} = 0$. **Thus for all $t \in R$ with t>0, $N_1^*(x,y,t) = 0 \Leftrightarrow$ x, y are linearly dependent.**

N3: As $\|x,y\|^*_{\alpha}$ is invariant under any permutation of x, y, it follows that N_1^* (x,y,t) is in variant under any permutation.

N4: For all $t \in R$ with t > 0 and c ≠ 0, $c \in F$, we have

$$N_1^*(x,cy,t) = \inf\{\alpha \in (0,1] : \|x,cy\|^*_{\alpha} \le t\} = \inf\{\alpha \in (0,1] : |c|\|x,y\|^*_{\alpha} \le t\} = \inf\{\alpha \in (0,1] : \|x,y\|^*_{\alpha} \le \frac{t}{|c|}\}$$

$$= N_1^*(x,y,\frac{t}{|c|}) \forall x,y \in U$$

N5: We have to show that,

$\forall s,t \in R and \forall x,y,u \in U, N_1^*(x,y+u,s+t) \le \max\{N_1^*(x,y,s).N_1^*(x,u,t)\}$.

Suppose that $\forall$ s ,$t \in$ R and $\forall$ $x, y, u \in$ U, $N_1^*(x, y+u, s+t)$ >max{$N_1^*(x,y,s)$, $N_1^*(x,u,t)$}.

Choose k such that $N_1^*(x,y+u,s+t)$>k>max{$N_1^*(x,y,s)$,$N_1^*(x,u,t)$}.

Now ,

$N_1^*(x,y+u,s+t) > k \Rightarrow \inf\{\alpha \in (0,1] : \|x,y+u\|^*_{\alpha} \le s+t\} > k \Rightarrow \|x,y+u\|^*_k \le s+t \Rightarrow \|x,y\|^*_k + \|x,u\|^*_k > s+t$

Again

$k > \max\{N_1^*(x,y,s), N_1^*(x,u,t)\} \Rightarrow k > N_1^*(x,y,s) \& k > N_1^*(x,u,t) \Rightarrow \|x,y\|^*_k \le s \& \|x,u\|^*_k \le t \Rightarrow \|x,y\|^*_k + \|x$
$\le s+t$.

Thus $s+t < \|x,y\|^*_k + \|x,u\|^*_k \le s+t$,which is a contradiction . Hence

$N_1^*(x,y+u,s+t) \le \max\{N_1^*(x,y,s), N_1^*(x,u,t)\}$.

N6: Let $x,y \in U, \alpha \in (0,1)$. Now $t > \|x,y\|^*_{\alpha} \Rightarrow N_1^*(x,y,t) = \inf\{\beta \in (0,1] : \|x,y\|^*_{\beta} \le t\} \le \alpha$. So $\lim_{t\to\infty} N_1^*(x,y,t) = 0$. Next we verify that $N_1^*(x,y,t)$ is a non-increasing function of $t \in R$. If $t_1 < t_2 \le 0, then N_1^*(x,y,t_1) = N_1^*(x,y,t_2) = 1, \forall x,y \in U$. If $0 < t_1 < t_2$ then

$\{\alpha \in (0,1] : \|x,y\|^*_{\alpha} \le t_1\} \subseteq \{\alpha \in (0,1] : \|x,y\|^*_{\alpha} \le t_2\} \Rightarrow \inf\{\alpha \in (0,1] : \|x,y\|^*_{\alpha} \le t_1\} \ge \inf\{\alpha \in (0,1] : \|x,y\|^*_{\alpha} \le t_2\} =$

$N_1^*(x,y,t_1) \ge N_1^*(x,y,t_2)$

Thus $N_1^*(x,y,t)$ is a non-increasing function of $t \in R$ and N_1^* is a fuzzy anti-2-norm on U.

For each $x \neq \underline{0}, y \neq \underline{0}, \|x,y\|^*_{\alpha} > 0$. Thus $\exists$ $r = r$ (x, y) >0 such that

$$\|x,y\|_{\alpha}^{*} \geq r(x,y) > 0 \Rightarrow r(x,y), \forall \alpha \in (0,1] \Rightarrow \inf\{\alpha \in (0,1] : \|x,y\|_{\alpha}^{*} \leq t\} = 1 \Rightarrow N_1^*(x,y,t) = 1.$$

Definition 1.9:

Let (U, N^*) be a Fa-2-NLS. A sequence $\{x_n\}$ in U is said to be *convergent* to $x \in U$ if given $t > 0$, $0<r<1$, there exists an integer $n_0 \in N$ such that $N^*(x_n - x, y, t) < r$, for all $n \geq n_0$.

Definition 1.10:

Let (U, N^*) be a Fa-2-NLS. A sequence $\{x_n\}$ in U is said to be a *Cauchysequence* if given $t>0$, $0<r<1$, there exists an integer $n_0 \in N$ such that

$N^*(x_{n+p} - x_n, y, t) < r$, for all $n \geq n_0$, $p=1,2,3,\ldots$.

Definition 1.11:

Let (U, N^*) be a Fa-2-NLS. A subset B of U is said to be *closed* iff or any sequence $\{x_n\}$ in B converges to $x \in B$, that is $\lim_{n\to\infty} N^*(x_n - x, y, t) = 0, \forall t > 0$ implies that $x \in B$.

Definition 1.12:

Let (U, N^*) be a Fa-2-NLS. A subset W of U is said to be the *closure* of $B \subset \underline{W}$ if for any $w \in W$, there exists a sequence $\{x_n\}$ in B such that

$\lim_{n\to\infty} N^*(x_n - x, y, t) = 0, \forall t \in R^+$, we denote the set W by B.

Definition 1.13:

A subset B of a Fa-2-NLS (U, N^*) is said to be *bounded* if and only if there exists $t>0$ and $0<r<1$ such that $N^*(x, y, t) < r$, $\forall$ $x, y \in B$.

Definition 1.14:

. A subset B of a Fa-2-NLS (U, N^*) is said to be *compact* if any sequence $\{x_n\}$ in B has a subsequence converging to an element of B

.Definition 1.15:

The fuzzy anti 2-normed linear space (P, N^*) in which every Cauchy sequence converges is called a complete fuzzy anti 2-normed linear space. The fuzzy anti 2-normed linear space (P, N^*) is a fuzzy anti 2-Banach space with respect to α anti-2-norm if it is a complete fuzzy anti 2-normed linear with respect to α anti-2-norm.

Definition 1.16.

A fuzzy anti 2-linear operator T is a function from $A \times B$ to $C \times D$ where A, B are subspaces of fuzzy anti 2-normed linear space (X, N_1^*) and C, D are subspaces of fuzzy anti 2-normed linear sp1ace (Y, N_2^*) such that

$$T(x_1 + x, x_2 + x') = T(x_1, x_2) + T(x_1, x') + T(x, x_2) + T(x, x')$$

$T(\alpha x_1, \beta x_2) = \alpha\beta T(x_1, x_2)$ **where** $\alpha, \beta \in (0,1)$

Definition 1.17.

Let T be a fuzzy anti 2-linear map from $A \times B$ to $C \times D$ where A, B are subspaces of (X, N_1^*) and C, D are subspaces of (Y, N_2^*) then it is said to be fuzzy anti 2-continuous at $(x_0, x_0') \in A \times B$ if for given $\varepsilon > 0, \alpha \in (0,1), \exists \delta = \delta(\alpha, \varepsilon) > 0,$

$\beta = \beta(\alpha, \varepsilon) \in (0,1)$ **such that** $\forall (x, x^{'}) \in A \times B$. $N_1^*[(x, x^{'}) - (x_0, x_0^{'}), \delta] > \beta$

$\Rightarrow N_2^*[T(x, x^{'}) - T(x_0, x_0^{'}), \varepsilon] < \alpha$

If it is fuzzy anti 2-continuous at each point of $A \times B$ **then T is *fuzzy anti 2-continuous* on** $A \times B$.

C

CONCLUSION

From this we learn the concept of Fuzzy anti -2 normed linear space and how it is applied in some concepts of mathematics.

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In fuzzy anti - 2 normed linear space we can apply all basic concepts in mathematics and we can show many important results through it.

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