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FUZZY ANTI 2 NORMED LINEAR SPACE

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ABSTRACT

In this paper , we study fuzzy anti 2-norm on a linear space and some results are introduced in fuzzy anti 2-norm on a linear space. We shall introduce the notion of convergent sequence, Cauchy sequence in fuzzy anti 2-normed linear space also introduce the concept of compact subset and bounded subset in fuzzy anti 2-normed linear space. Then we introduced closed graph theorem and proved it in Fa 2-NLS and Riesz theorem in Fa n-NLS. Then we study the concept of t –best coapproximation in fuzzy anti 2-normed linear space.

Keywords: Fuzzy anti 2-norm, convergent sequence, Cauchy sequence, Fuzzy anti 2-linear operator.

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1. BASIC DEFINITIONS IN FUZZY ANTI-2 NORMED LINEAR SPACE

Definition 1.1:

A function $\sigma : A \to [0,1]$ is called a membership function on *A*. The set *A* together with a membership function σ is called a fuzzy set and is denoted by (A, σ). we can also denote this *A* as a *fuzzy set* or σ is a fuzzy set.

Definition 1.2:

A *normed linear space* is a vector space *X* and a non-negative valued mapping $||\cdot||$ on *X*, called the norm , which satisfies the properties

1. **||x||=0 if and only if x=0.**
2. **||a x|| =|a| ||x||, for all scalars a.**
3. **||x+y|| ≤ ||x|| + ||y||**

Definition 1.3:

Let *X* be a linear space over a real field *F* (field of real/complex numbers). A fuzzy subset *N* of $X \times R$ is called a fuzzy norm on *X* if the following conditions are satisfied for all $x, y \in X$

F1: For all $t \in R$ with $t \le 0$, $N(x, t) = 0$

F2: For all $t \in R$ with $t>0$, $N(x, t) = 1$ if and only if $x = \underline{0}$

F3: For all $t \in R$ with $t>0$, $N(cx, t) = N(x, \frac{t}{|c|})$ if $c \neq 0$, $c \in F$

F4: For all $s, t \in R$, $N(x + y, s + t) \geq \min\{N(x, s), N(y, t)\}$

F5: $N(x, .)$ is a non-decreasing function of R and $\lim_{t \to \infty} N(x,t) = 1$.

Then *N* is said to be a *fuzzy norm on a linear spaceX* and the pair (*X*, *N*) is said to be a Fuzzy normed linear space (FNLS).
The following condition of fuzzy norm N will be required later on

F6: $N(x, t) > 0$, $\forall t > 0$ implies $x = \underline{0}$

Definition 1.4:

Let *U* be a linear space over a real field *F*. A fuzzy subset N^* of *X*×*R* such that for all *x, u* ∈*U* and c ∈ F

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N^*1: **For all $t \in R$ with $t \leq 0$, $N^*(x, t)=1$,**

N^*2: **For all $t \in R$ with $t>0$ $N^*(x,t) = 0 \Leftrightarrow x = \underline{0}$**

N^*3: **For all $t \in R$ with $t>0$, $N^*(cx,t) = N^*(x, \frac{t}{|c|})$ if $c \neq 0, c \in F$**

N^*4: **For all $s, t \in R$, $N^*(x+u, s+t) \leq \max\{N^*(x, s), N^*(u, t)\}$**

N^*5: **$N^*(x, t)$ is a non-increasing function of $t \in R$ and $\lim_{t\to\infty} N^*(x, t) = 0$.Then $N^*$is said to be a *fuzzy anti norm on a linear space U* and the pair (U, N^*) is called a fuzzy anti normed linear space (briefly Fa-NLS).**

N^*6: **For all $t \in R$ with $t>0$, $N^*(x,t) < 1$ implies $x = \underline{0}$**

Definition 1.5:

Let U be a linear space over a real field F. A fuzzy subset N^* of $U \times U \times R$ such that for all $x, y, u \in U$

N1: For all $t \in R$ with $t \leq 0$, $N^*(x, y, t)=1$.

N2: For all $t \in R$ with $t>0$, $N^*(x, y, t) = 0$ if and only if x, y are linearly dependent.

N3: $N^*(x, y, t)$ is invariant under any permutation of x, y.

N4: For all $t \in R$ with $t>0$, $N^*(x, cy, t) = N^*(x, y, \frac{t}{|c|})$ if $c \neq 0, c \in F$.

N5: For all $s, t \in R$, $N^*(x, y+u, s+t) \leq \max\{N^*(x, y, s), N^*(x, u, t)\}$.

N6: $N^*(x, y, t)$ is a non-increasing function of $t \in R$ and $\lim_{t\to\infty} N^*(x, y, t) = 0$.

Then $N^*$is said to be a fuzzy anti-2-norm on a linear space U and the pair (U, N^*) is called a *fuzzy anti-2-normed linear space*.(briefly Fa-2-NLS).

The following condition of fuzzy anti-2-norm N^*will be required later on.

N7: For all $t \in R$ with $t>0$, $N^*(x, y, t) < 1$, implies that x, y are linearly dependent.

Example :

Let $(U, \bullet, \bullet)$ be a 2-normed linear space .Define

$$N^*(x,y,t) = \frac{\|x,y\|}{t + \|x+y\|}, \text{ when } t>0,\ t \in R,\ x, y \in U$$

$$= 1, \text{ when } t \leq 0,\ t \in R,\ x, y \in U.$$

Then (U, N^*) is an Fa-2-NLS.

Proof:

Now we have to show that $N^*(x, y, t)$ is a fuzzy anti-2-norm in U

N1: For all $t \in R$ with $t \leq 0$, we have by definition $N^*(x, y, t)=1$.

N2: For all $t \in R$ with $t>0$ N^*~~(x, y ,t)~~$=0 \Leftrightarrow \frac{\|x,y\|}{t+\|x,y\|} = 0 \Leftrightarrow \|x,y\| = 0 \Leftrightarrow$ x, y are linearly independent.

N3: As $\|x,y\|$ is invariant under any permutation of x, y it follows that $N^*(x,y,t)$ is invariant under any permutation of x, y.

N4: For all $t \in R$ with $t>0$ and $c \neq 0$, $c \in F$, we get

$$N^*(x,cy,t)=\frac{\|x,cy\|}{t+\|x,cy\|}=\frac{|c|\|x,y\|}{t+|c|\|x,y\|}=\frac{\|x,y\|}{t/|c|+\|x,y\|}=N^*(x,y,\ t/|c|)$$

N5: For all $s,t\in R$ and $x,y,u\in U$. We have to show that $N^*(x,y+u,s+t)\le\max\{N^*(x,y,s),\ N^*(x,u,t)\}$. If (a) $s+t<0$ (b) $s=t=0$ (c) $s+t>0$; $s>0$, $t<0$; $s<0$, $t>0$, then in these cases the relation is obvious . If (d) $s>0$, $t>0$, $s+t>0$. Then assume that

$$N^*(x,y,s)\le N^*(x,u,t)\Rightarrow\frac{\|x,y\|}{s+\|x,y\|}\le\frac{\|x,u\|}{t+\|x,u\|}\Rightarrow\|x,y\|(t+\|x,u\|)\le\|x,u\|(s+\|x,y\|)$$

$$\Rightarrow t\|x,y\|\le s\|x,u\|\ ....(1)$$

Now,

$$\frac{\|x,y+u\|}{s+t+\|x,y+u\|}-\frac{\|x,u\|}{t+\|x,u\|}\le\frac{\|x,y\|+\|x,u\|}{s+t+\|x,y\|+\|x,u\|}-\frac{\|x,u\|}{t+\|x,u\|}=\frac{t\|x,y\|-s\|x,u\|}{(s+t+\|x,y\|+\|x,u\|)(t+\|x,u\|)}$$

By using equation (1), we get $\frac{\|x,y+u\|}{s+t+\|x,y+u\|}\le\frac{\|x,u\|}{t+\|x,u\|}$ **, similarly** $\frac{\|x,y+u\|}{s+t+\|x,y+u\|}\le\frac{\|x,y\|}{s+\|x,y\|}$

Hence $N^*(x,y+u,s+t)\le\max\{N^*(x,y,s),N^*(x,u,t)\}$

N6: If $t_1<t_2\le 0$ then we have $N^*(x,y,t_1)=N^*(x,y,t_2)=1$. If $0<t_1<t_2$ then

$$\frac{\|x,y\|}{t_1+\|x,y\|}-\frac{\|x,y\|}{t_2+\|x,y\|}=\frac{\|x,y\|(t_2-t_1)}{(t_1+\|x,y\|)(t_2+\|x,y\|)}>0\Rightarrow N^*(x,y,t_1)\ge N^*(x,u,t_2)$$

Thus $N^*(x,y,t)$ is a non-increasing function of $t\in R$. Again

$$\lim_{t\to\infty}N^*(x,y,t)=\lim_{t\to\infty}\frac{\|x,y\|}{t+\|x,y\|}=0\ \forall x,y\in U\ .$$

Hence (U,N^*) is an Fa-2 NLS.

Definition 1.6:

Let N^* be a fuzzy anti-2-norm on U satisfying N7. Define $\|x,y\|_\alpha^*=\inf\{t>0: N^*(x,y,t)<\alpha$, $\alpha\in(0,1]\}$.

Lemma 1.7:

Let (U,N^*) be a Fa-2-NLS. For each $\alpha\in(0,1]$ and $x,y,u\in U$. Then we ha

(i) $\|x,y\|_{\alpha_1}^*\ge\|x,y\|_{\alpha_2}^*$ **for $0<\alpha_1<\alpha_2\le 1$,**

(ii) $\|x,cy\|_\alpha^*=|c|\|x,y\|_\alpha^*$ **for any scalar c,**

(iii) $\|x,y+u\|_\alpha^*\le\|x,y\|_\alpha^*+\|x,u\|_\alpha^*$

Proof:

(i) For $0<\alpha1<\alpha2\le 1$, we note that

$\inf\{t>0$: $N^*(x,y,t)<\alpha_1\}\ge\inf\{N^*(x,y,t)<\alpha_1\}\Rightarrow\|x,y\|_{\alpha_1}^*\ge\|x,y\|_{\alpha_2}^*$

(ii) For any scalar c and for all $\alpha\in(0,1]$,

$\|x, cy\|_\alpha^*$ = **inf{t>0: N^* (x,cy,t)<α, α∈(0,1]}=inf{t>0:** $N^*(x, y, \frac{t}{|c|}) < \alpha, \alpha \in (0,1]\}$

$= |c| \inf\{t > 0 : N^*(x, y, t) < \alpha, \alpha \in (0,1]\} = |c| \|x, y\|_\alpha^*$

(iii) For any α ∈ (0,1],

$$\|x, y\|_\alpha^* + \|x, u\|_\alpha^* = \inf\{t > 0 : N^*(x.y, t) < \alpha\} + \inf\{s > 0 : N^*(x, u, s) < \alpha\}$$

$$\geq \inf\{s + t > 0 : N^*(x, y, t) < \alpha, N^*(x, u, s) < \alpha\} = \|x, y + u\|_\alpha^*$$

Theorem 1.8.

Let {$\|\bullet,\bullet\|_\alpha^*$: α∈(0,1]}be a decreasing family of 2-norms on a linear space *U*. Now define a function

$N_1^* : U \times U \times R \to [0,1]$ **as**

$$N_1^*(x, y, t) = \inf\{\alpha \in (0,1] : \|x, y\|_\alpha^*\}, \text{ when } (x, y, t) \neq 0$$

$$= 1, \text{ when } (x, y, t) = 0$$

Then,

(a) *N is a fuzzy anti-2-norm on *U*.**

(b) For each *x*, *y* ∈*U*, ∃*r* = *r*(*x*, *y*)>0 such that $N_1^*(x, y, t) = 1$

***Proof*.**

(a) Now we have to show that $N^*$is a fuzzy anti-2-norm on *U*.

N1: (i) For all *t* ∈*R* with *t*<0,{α∈(0,1]: $\|x, y\|_\alpha^* \leq t$**}=Φ,∀ *x*, *y*∈*U*, we have**

$N_1^*(x, y, t) = \inf\{\alpha \in (0,1] : \|x, y\|_\alpha^* \leq t\} = 1.$

(ii) For *t*= 0 and $x \neq \underline{0}$, $y \neq \underline{0}$, **{α ∈ (0,1]:** $\|x, y\|_\alpha^* \leq t$**}= Φ, ∀ *x*, *y*∈ *U*, we have** $N_1^*(x, y, t) = 1$.

(iii) For*t*=0 and $x \neq \underline{0}$, $y \neq \underline{0}$, **then from the definition** $N_1^*(x, y, t) = 1$.

Thus for all *t*∈*R* with *t*≤0, $N_1^*(x, y, t) = 1$, **∀*x*, *y*∈*U*.**

N2: For all *t* ∈*R* with *t*>0, $N^*(x, y, t) = 0$. **Choose any ε ∈ (0,1). Then for any *t*>0, ∃α_1∈(ε,1] such that** $\|x.y\|_{\alpha_1}^* \leq t$ **and hence** $\|x, y\|_\varepsilon^* \leq t$. **Since t>0 is arbitrary, this implies that** $\|x, y\|_\varepsilon^* = 0$ **then x, y are linearly independent. If x, y are linearly dependent then for t>0,**

$N_1^*(x, y, t) = \inf\{\alpha \in (0,1] : \|x, y\|_\alpha^* \leq t\} = 0$. **Thus for all** $t \in R$ **with t>0,** $N_1^*(x, y, t) = 0 \Leftrightarrow$ **x, y are linearly dependent.**

N3: As $\|x, y\|_\alpha^*$ **is invariant under any permutation of *x*, *y*, it follows that** N_1^* **(*x*,*y*,*t*) is in variant under any permutation.**

N4: For all $t \in R$ **with t > 0 and c ≠ 0,** $c \in F$, **we have**

$$N_1^*(x,cy,t)=\inf\{\alpha\in(0,1]:\|x,cy\|_\alpha^*\le t\}=\inf\{\alpha\in(0,1]:|c|\|x,y\|_\alpha^*\le t\}=\inf\{\alpha\in(0,1]:\|x,y\|_\alpha^*\le\frac{t}{|c|}\}$$

$$=N_1^*(x,y,\frac{t}{|c|})\forall x,y\in U$$

N5: We have to show that,
$\forall s,t\in R and \forall x,y,u\in U, N_1^*(x,y+u,s+t)\le\max\{N_1^*(x,y,s).N_1^*(x,u,t)\}$.

Suppose that ∀ s ,t∈ R and ∀ x, y, u∈ U, $N_1^*(x, y+u, s+t) > \max\{N_1^*(x,y,s), N_1^*(x,u,t)\}$. Choose k such that $N_1^*(x,y+u,s+t)>k>\max\{N_1^*(x,y,s),N_1^*(x,u,t)\}$.
Now ,

$$N_1^*(x,y+u,s+t)>k\Rightarrow\inf\{\alpha\in(0,1]:\|x,y+u\|_\alpha^*\le s+t\}>k\Rightarrow\|x,y+u\|_k^*\le s+t\Rightarrow\|x,y\|_k^*+\|x,u\|_k^*>s+t$$

Again

$$k>\max\{N_1^*(x,y,s),N_1^*(x,u,t)\}\Rightarrow k>N_1^*(x,y,s)\,\&\,k>N_1^*(x,u,t)\Rightarrow\|x,y\|_k^*\le$$
$$\le s+t.$$

Thus $s+t<\|x,y\|_k^*+\|x,u\|_k^*\le s+t$ **,which is a contradiction . Hence**

$N_1^*(x,y+u,s+t)\le\max\{N_1^*(x,y,s),N_1^*(x,u,t)\}$.

N6: Let $x,y\in U,\alpha\in(0,1)$. **Now** $t>\|x,y\|_\alpha^*\Rightarrow N_1^*(x,y,t)=\inf\{\beta\in(0,1]:\|x,y\|_\beta^*\le t\}\le\alpha$. **So** $\lim_{t\to\infty}N_1^*(x,y,t)=0$. **Next we verify that** $N_1^*(x,y,t)$ **is a non-increasing function of** $t\in R$. **If** $t_1<t_2\le 0, then N_1^*(x,y,t_1)=N_1^*(x,y,t_2)=1,\forall x,y\in U$. **If** $0<t_1<t_2$ **then**

$$\{\alpha\in(0,1]:\|x,y\|_\alpha^*\le t_1\}\subseteq\{\alpha\in(0,1]:\|x,y\|_\alpha^*\le t_2\}\Rightarrow\inf\{\alpha\in(0,1]:\|x,y\|_\alpha^*\le t_1\}\ge\inf\{\alpha\in(0,1]:\|x,y\|_\alpha^*\le t_2\}\Rightarrow$$
$$N_1^*(x,y,t_1)\ge N_1^*(x,y,t_2)$$

Thus $N_1^*(x,y,t)$ **is a non-increasing function of** t∈ R **and** N_1^* **is a fuzzy anti-2-norm on U.**

For each $x\ne\underline{0},y\ne\underline{0},\|x,y\|_\alpha^*>0$ **. Thus ∃** $r=r(x,y)>0$ **such that**

$$\|x,y\|_\alpha^*\ge r(x,y)>0\Rightarrow r(x,y),\forall\alpha\in(0,1]\Rightarrow\inf\{\alpha\in(0,1]:\|x,y\|_\alpha^*\le t\}=1\Rightarrow N_1^*(x,y,t)=1.$$

Definition 1.9:
Let (U,N^*) be a Fa-2-NLS. A sequence $\{x_n\}$ in U is said to be *convergent* to x∈U if given $t>0$, $0<r<1$, there exists an integer n_0∈N such that $N^*(x_n-x,y,t)<r$, for all $n\ge n_0$.

Definition 1.10:
Let (U,N^*) be a Fa-2-NLS. A sequence $\{x_n\}$ in U is said to be a *Cauchysequence* if given $t>0$, $0<r<1$, there exists an integer n_0∈N such that

$N^*(x_{n+p}-x_n,y,t)<r$, for all $n\ge n_0$, p=1,2,3,….

Definition 1.11:
Let (U,N^*) be a Fa-2-NLS. A subset B of U is said to be *closed* iff or any sequence $\{x_n\}$ in B converges to x∈B, that is $\lim_{n\to\infty}N^*(x_n-x,y,t)=0,\forall t>0$ implies that x∈B.

Definition 1.12:
Let (U,N^*) be a Fa-2-NLS. A subset W of U is said to be the *closure* of $B\subset\underline{W}$ if for any w∈W, there exists a sequence $\{x_n\}$ in B such that

$\lim_{n\to\infty} N^*(x_n - x, y, t) = 0, \forall t \in R^+$, **we denote the set** W **by** B.

Definition 1.13:

A subset B **of a Fa-2-NLS** (U, N^*) **is said to be** ***bounded*** **if and only if there exists** $t>0$ **and** $0<r<1$ **such that** $N^*(x, y, t)<r, \forall x, y \in B$.

Definition 1.14:

. A subset B **of a Fa-2-NLS** (U, N^*) **is said to be** ***compact*** **if any sequence** $\{x_n\}$ **in** B **has a subsequence converging to an element of** B

.Definition 1.15:

The fuzzy anti 2-normed linear space (P, N^*) **in which every Cauchy sequence converges is called a complete fuzzy anti 2-normed linear space. The fuzzy anti 2-normed linear space** (P, N^*) **is a fuzzy anti 2-Banach space with respect to α anti-2-norm if it is a complete fuzzy anti 2-normed linear with respect to α anti-2-norm.**

Definition 1.16.

A fuzzy anti 2-linear operator T is a function from $A \times B$ **to** $C \times D$ **where A, B are subspaces of fuzzy anti 2-normed linear space** (X, N_1^*) **and C, D are subspaces of fuzzy anti 2-normed linear sp1ace** (Y, N_2^*) **such that**

$$T(x_1 + x, x_2 + x') = T(x_1, x_2) + T(x_1, x') + T(x, x_2) + T(x, x')$$

$T(\alpha x_1, \beta x_2) = \alpha\beta T(x_1, x_2)$ **where** $\alpha, \beta \in (0,1)$

Definition 1.17.

Let T be a fuzzy anti 2-linear map from $A \times B$ **to** $C \times D$ **where A, B are subspaces of** (X, N_1^*) **and C, D are subspaces of** (Y, N_2^*) **then it is said to be fuzzy anti 2-continuous at** $(x_0, x_0') \in A \times B$ **if for given**

$\varepsilon > 0, \alpha \in (0,1), \exists \delta = \delta(\alpha, \varepsilon) > 0,$

$\beta = \beta(\alpha, \varepsilon) \in (0,1)$ **such that** $\forall (x, x') \in A \times B . N_1^*[(x, x') - (x_0, x_0'), \delta] > \beta$

$\Rightarrow N_2^*[T(x, x') - T(x_0, x_0'), \varepsilon] < \alpha$

If it is fuzzy anti 2-continuous at each point of $A \times B$ **then T is** ***fuzzy anti 2-continuous*** **on** $A \times B$.

CONCLUSION

From this we learn the concept of Fuzzy anti -2 normed linear space and how it is applied in some concepts of mathematics.

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In fuzzy anti - 2 normed linear space we can apply all basic concepts in mathematics and we can show many important results through it.

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